University of BRISTOL
School of Physics

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Quantum Conservation Laws

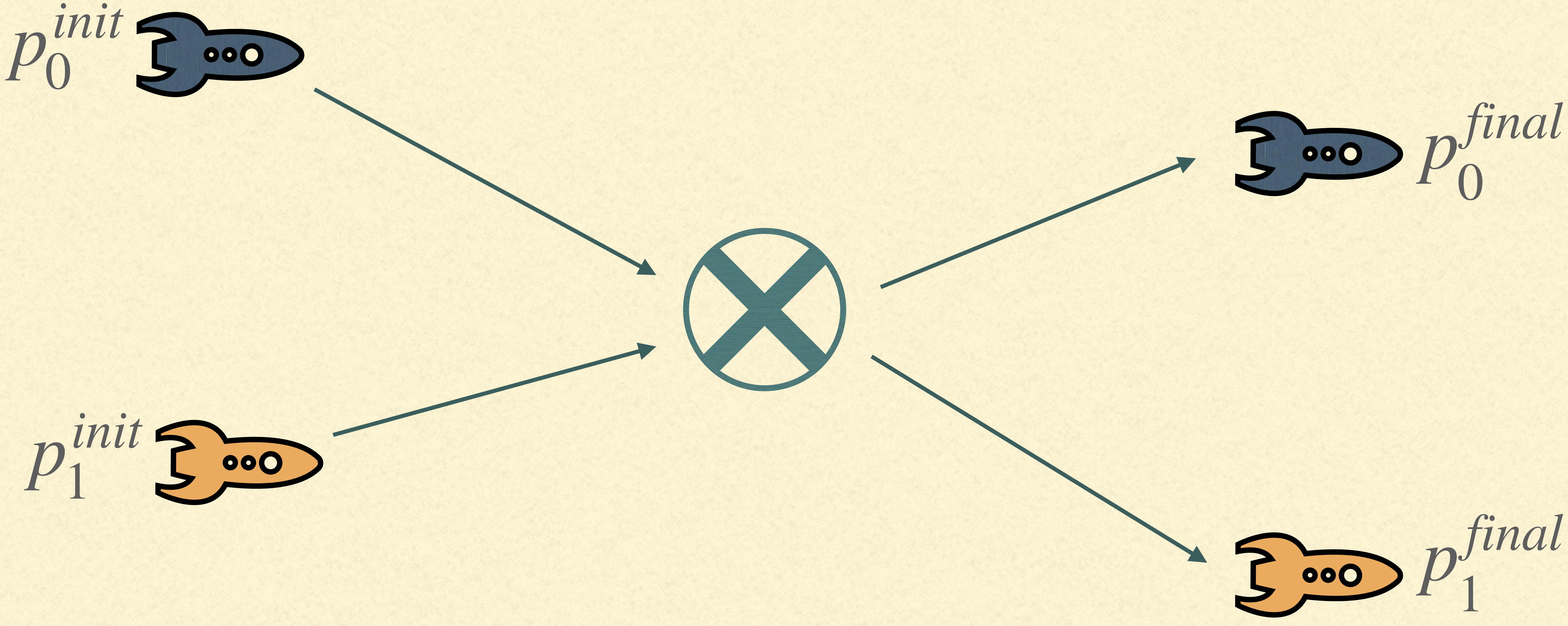
and how to fix them

Daniel Collins and Sandu Popescu

Example: Conservation of Energy

How high will the ball roll?

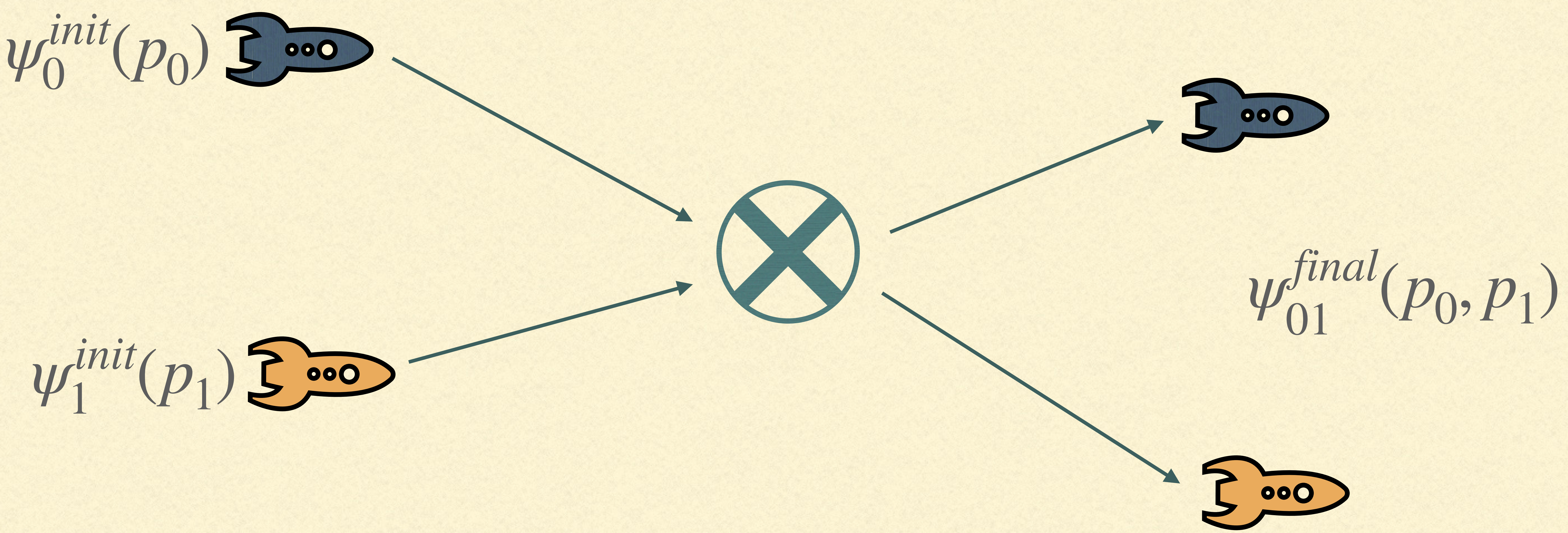
Classical Conservation Laws



Total Momentum Before = Total Momentum After

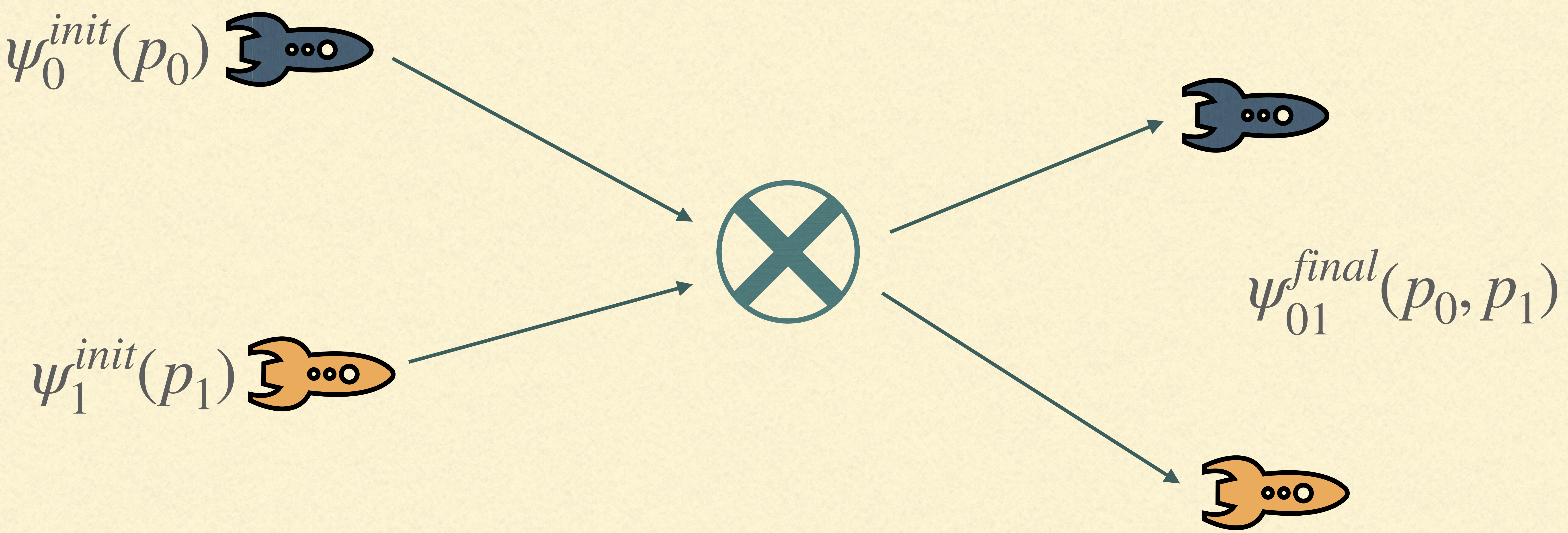
$$p_0^{init} + p_1^{init} = p_0^{final} + p_1^{final}$$

Quantum Conservation Laws



Distribution of Total Momentum Before = Distribution of Total Momentum After

Quantum Conservation Laws



Distribution of Total Momentum Before = Distribution of Total Momentum After

$$g(p_{tot}) = \int |\psi_0^{init}(p_{tot} - q)|^2 |\psi_1^{init}(q)|^2 dq = \int |\psi_{01}^{final}(p_{tot} - q, q)|^2 dq$$

Conservation in a Measurement

$$\frac{1}{\sqrt{2}}\left(|-1\rangle + |+1\rangle\right) \quad \xrightarrow{Prob = \frac{1}{2}} \quad |+1\rangle$$

$$\frac{1}{\sqrt{2}}\left(|-1\rangle + |+1\rangle\right) \quad \xrightarrow{Prob = \frac{1}{2}} \quad |-1\rangle$$

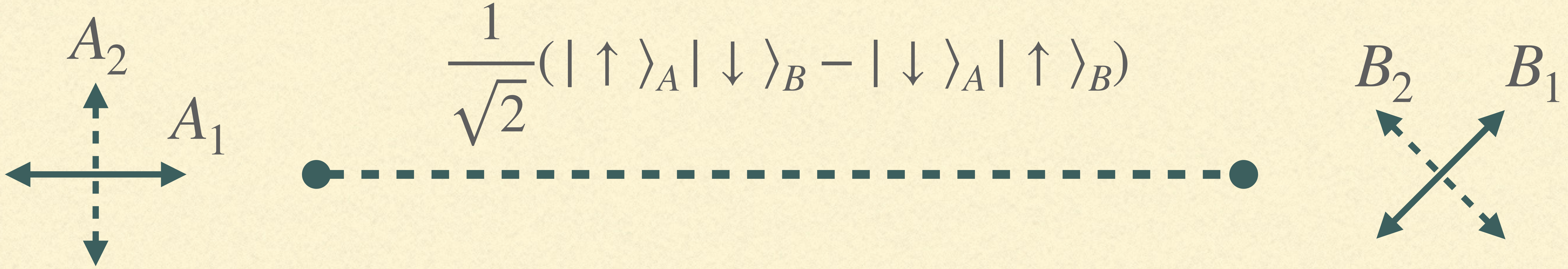
Conserved Quantity Jumps.

Jump Problem - Hidden Variables?

* Classically we simply improve our knowledge of a pre-existing state.

Jump Problem - Hidden Variables?

* Classically we simply improve our knowledge of a pre-existing state.
* Quantum Mechanically the outcome is not pre-existing (Bell Inequalities).



$$\frac{1}{\sqrt{2}}(|\uparrow\rangle_A|\downarrow\rangle_B - |\downarrow\rangle_A|\uparrow\rangle_B)$$

Jump Problem - Measuring Device?

* Does the jump come from the measuring device, or the human act of measurement?

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* Does the jump come from the measuring device, or the human act of measurement?
* No: the measuring interaction can leave the measuring device with the same value of the conserved quantity as it started.

Measurement Interaction

Define measurement of $\hat{\mathbf{L}}$, Angular Momentum on a circle:

$$|\hat{\mathbf{L}} = l_1\rangle_S |\hat{\mathbf{q}} = 0\rangle_M \rightarrow |\hat{\mathbf{L}} = l_1\rangle_S |\hat{\mathbf{q}} = 1\rangle_M$$
$$|\hat{\mathbf{L}} = l_2\rangle_S |\hat{\mathbf{q}} = 0\rangle_M \rightarrow |\hat{\mathbf{L}} = l_2\rangle_S |\hat{\mathbf{q}} = 2\rangle_M$$

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Measure System:

$$\left(\alpha |l_1\rangle_S + \beta |l_2\rangle_S\right) |0\rangle_M \rightarrow \alpha |l_1\rangle_S |1\rangle_M + \beta |l_2\rangle |2\rangle_M$$

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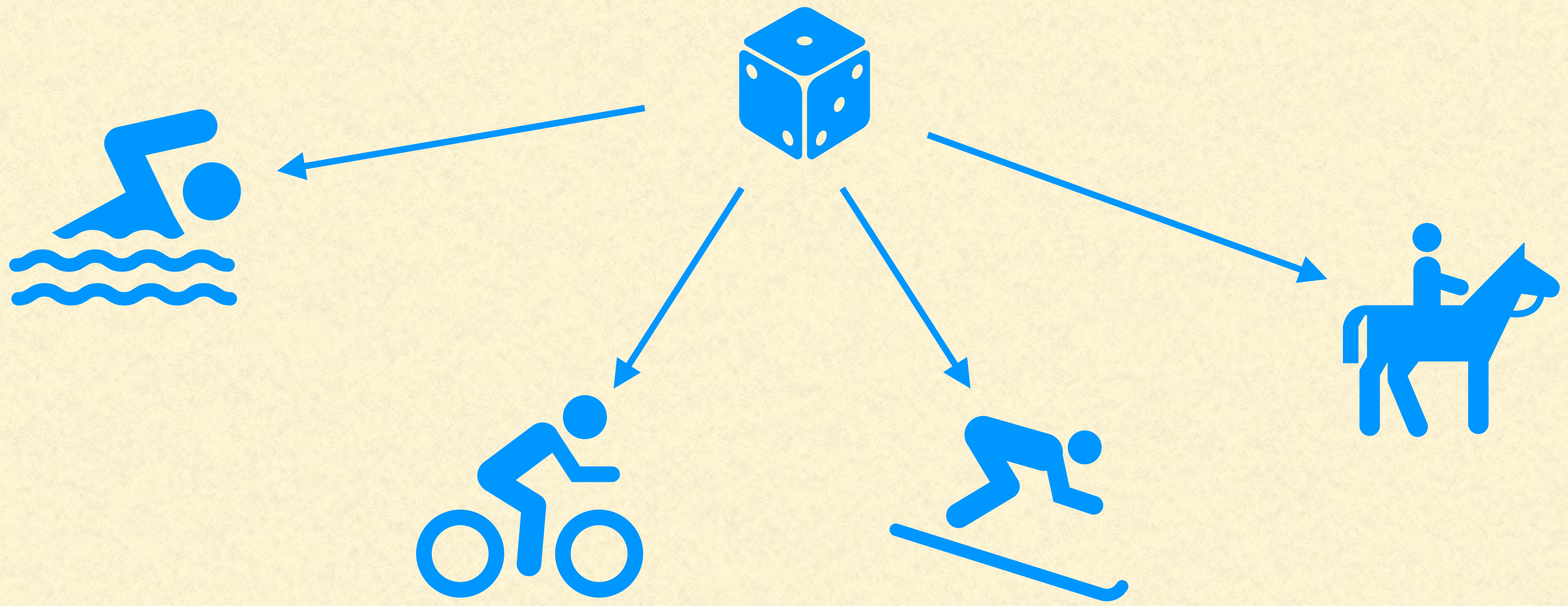
Read Pointer:

$$\begin{cases} |l_1\rangle_S |1\rangle_M \quad \text{with} \quad \text{probability} \quad |\alpha|^2 \\ |l_2\rangle_S |2\rangle_M \quad \text{with} \quad \text{probability} \quad |\beta|^2 \end{cases}$$

Jump Problem - Many Worlds?



* In Many Worlds the whole distribution is realised across the multiverse.
* So no jump at level of multiverse.

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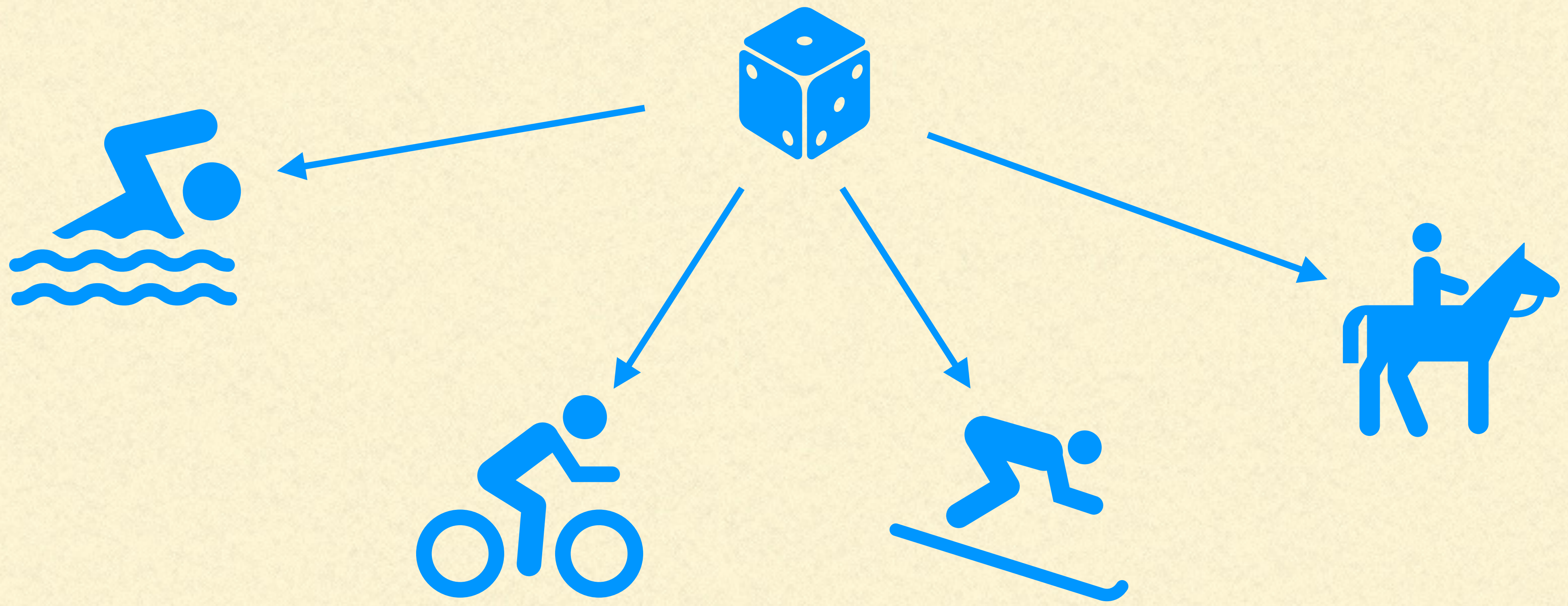


* In Many Worlds the whole distribution is realised across the multiverse.
* So no jump at level of multiverse.
* However we only see one world, and in our world there is a jump.

How can a Conserved Quantity Jump?

$$\left(\alpha|l_1\rangle_S+\beta|l_2\rangle_S\right)|0\rangle_M \xrightarrow{Prob=|\alpha|^2} |l_1\rangle_S|1\rangle_M$$

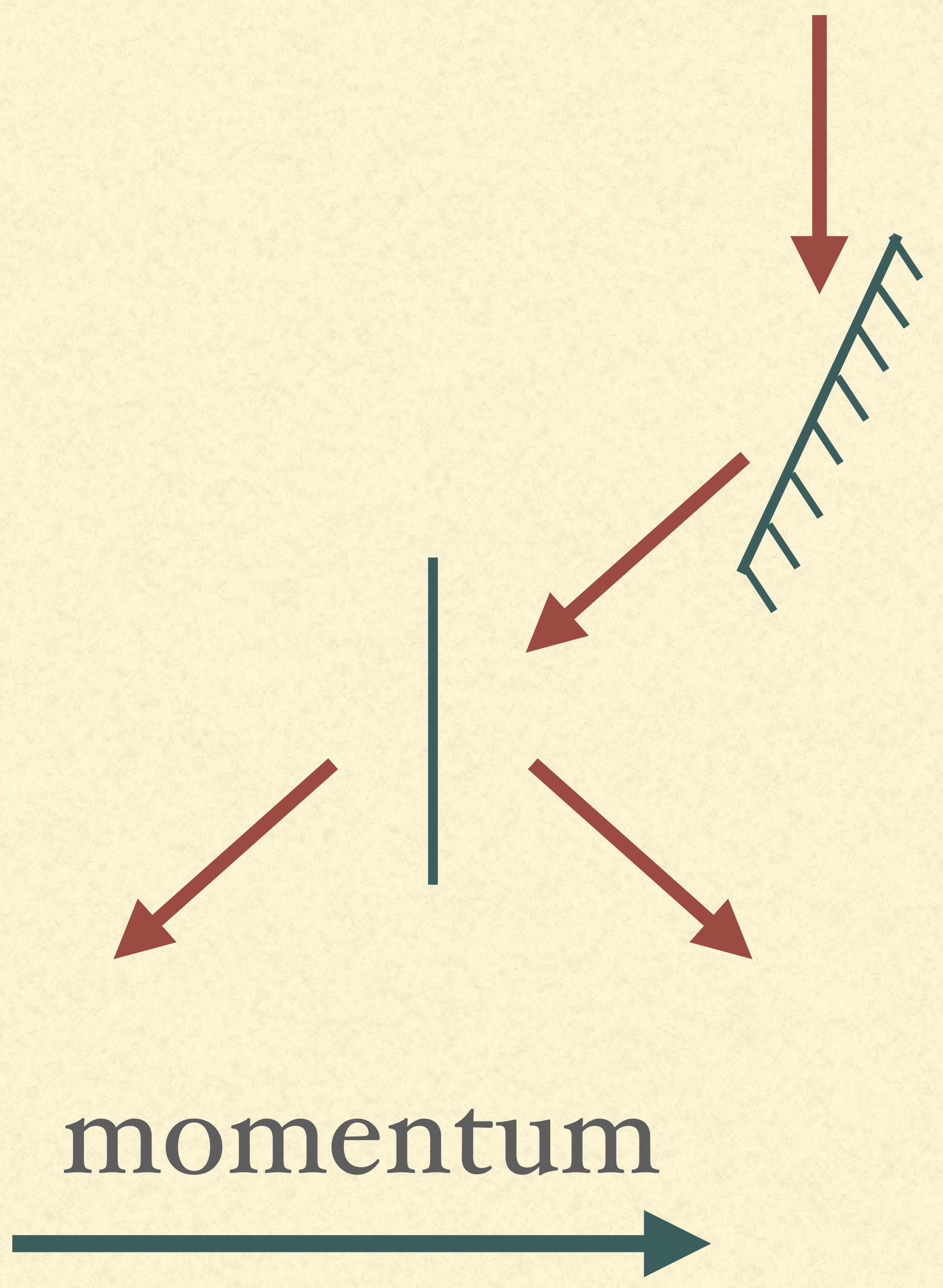
$$\left(\alpha|l_1\rangle_S+\beta|l_2\rangle_S\right)|0\rangle_M \xrightarrow{Prob=|\beta|^2} |l_2\rangle_S|2\rangle_M$$

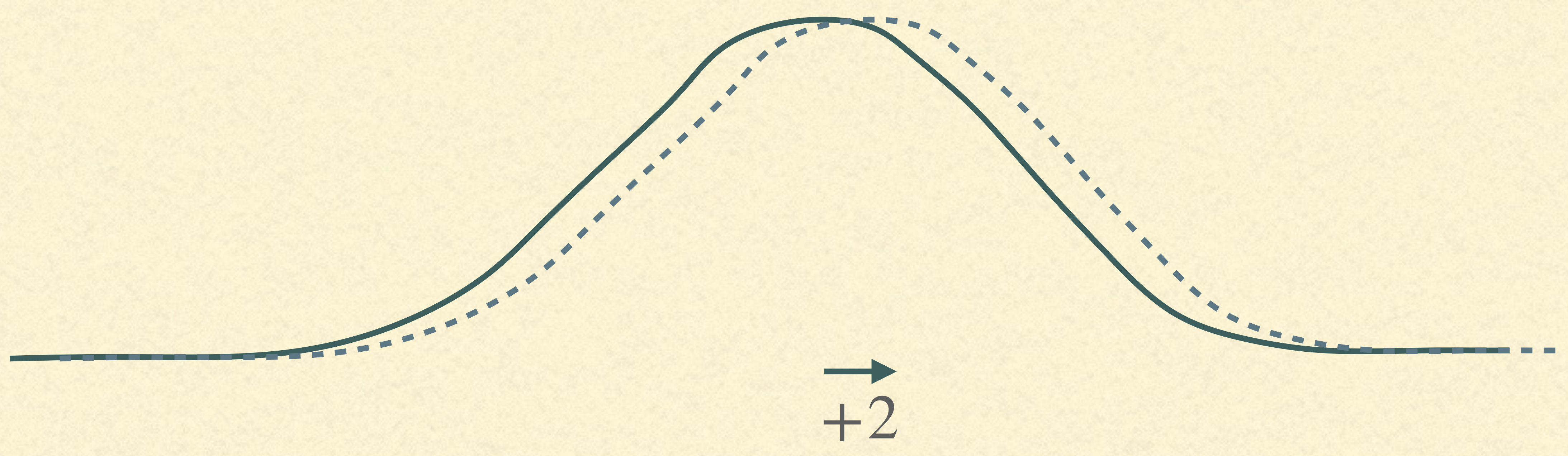
No hidden variable, not from measuring device, no help from many worlds.

The Preparation

Preparer (mirror and beamsplitter) starts in a state with a wide and smooth distribution of momentum.

$$|\Phi\rangle_P|0\rangle_S \rightarrow \frac{1}{\sqrt{2}}\left(|\Phi-1\rangle_P|+1\rangle_S + |\Phi+1\rangle_P|-1\rangle_S\right)$$





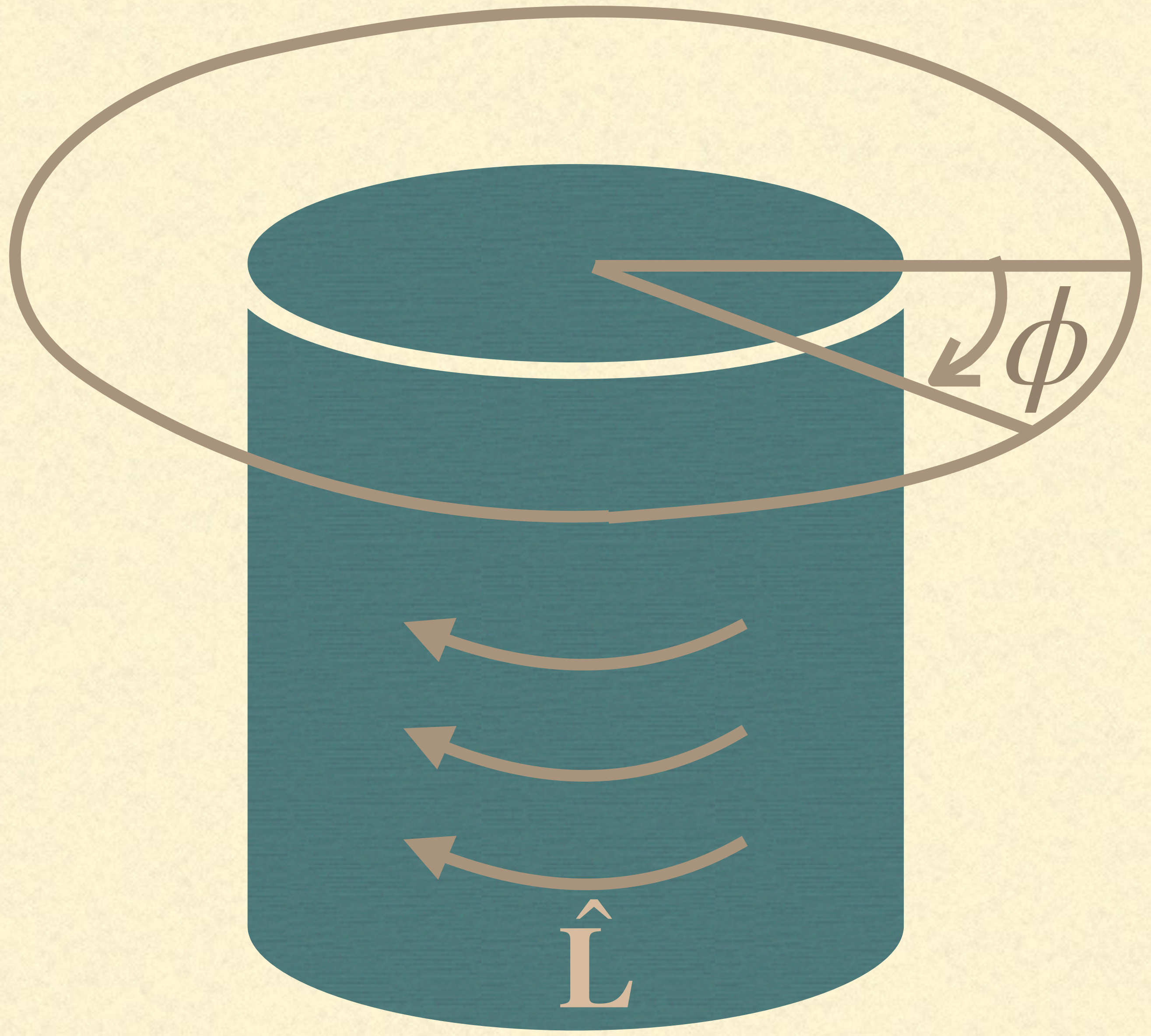
What if there is no Preparer ?

Suppose the state is precisely:

$$\cos\theta\,|\,l_1\rangle_S + e^{i\phi}\sin\theta\,|\,l_2\rangle_S$$

The Preparer has vanished.

The Meaning of ϕ



ϕ is an absolute angle, which has no meaning.

Frame of Reference

System with absolute angle:

$$\int \tilde{\Psi}(\phi_s)\,|\,\phi_s\rangle_S\, d\phi_s$$

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$$\int \tilde{\Psi}(\phi_s)\,|\,\phi_s\rangle_S \, d\phi_s$$

Relative to frame of reference F:

$$\int \tilde{\Phi}_f(\phi_f)\,|\,\phi_f\rangle_F \int \tilde{\Psi}(\phi_s - \phi_f)\,|\,\phi_s\rangle_S \, d\phi_s \, d\phi_f$$

Frame of Reference = Preparer

System with absolute angle:

$$\int \tilde{\Psi}(\phi_s) \,|\, \phi_s \rangle_S \, d\phi_s$$

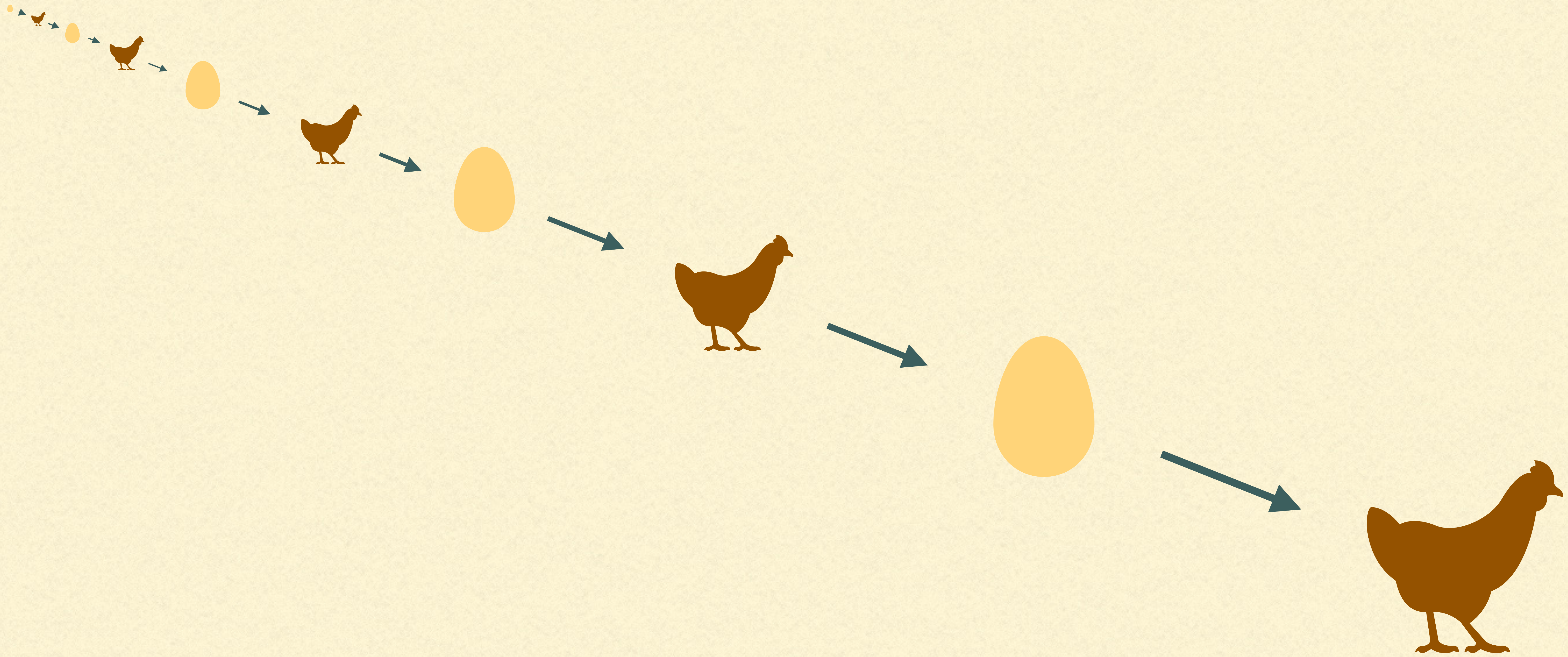
Relative to frame of reference F:

$$\int \tilde{\Phi}_f(\phi_f) \,|\, \phi_f \rangle_F \int \tilde{\Psi}(\phi_s - \phi_f) \,|\, \phi_s \rangle_S \, d\phi_s \, d\phi_f$$

Preparation in angular basis:

$$\int \tilde{\Phi}_p(\phi_p) \,|\, \phi_p \rangle_P \, d\phi_p \frac{1}{\sqrt{2\pi}} \int |\, \phi_s \rangle_S \, d\phi_s \rightarrow \int \tilde{\Phi}_p(\phi_p) \,|\, \phi_p \rangle_P \int \tilde{\Psi}(\phi_s - \phi_p) \,|\, \phi_s \rangle_S \, d\phi_s \, d\phi_p$$

Who Prepared the Preparer ?



The Grand-Preparer

Start: $|\Phi_g\rangle_G |0\rangle_P |0\rangle_S$.

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Preparation: $|l\rangle_G|0\rangle_P \to \sum_{k=-\infty}^{\infty} \Phi_p(k)|l-k\rangle_G|k\rangle_P$.

No Grand-Preparer

Start: $|\Phi_g\rangle_G|0\rangle_P|0\rangle_S$.

Preparation: $|l\rangle_G|0\rangle_P \to \sum_{k=-\infty}^{\infty} \Phi_p(k)|l-k\rangle_G|k\rangle_P$.

Time	Preparer $\mathbb{P}(\hat{\mathbf{L}}_p = l)$	Grand-Preparer $\mathbb{P}(\hat{\mathbf{L}}_g = k)$
Preparer Prepared	$\lvert\Phi_p(l)\rvert^2$	$\sum_{l=-\infty}^{\infty} \lvert\Phi_g(k+l)\rvert^2 \lvert\Phi_p(l)\rvert^2$
Measurement gives $\hat{\mathbf{L}}_s = l_0$	$\lvert\Phi_p(l+l_0)\rvert^2$	$\sum_{l=-\infty}^{\infty} \lvert\Phi_g(k+l)\rvert^2 \lvert\Phi_p(l)\rvert^2$

The Grand-Preparer is not required for our conservation law, only the Preparer.

Does Preparer exist in Nature?

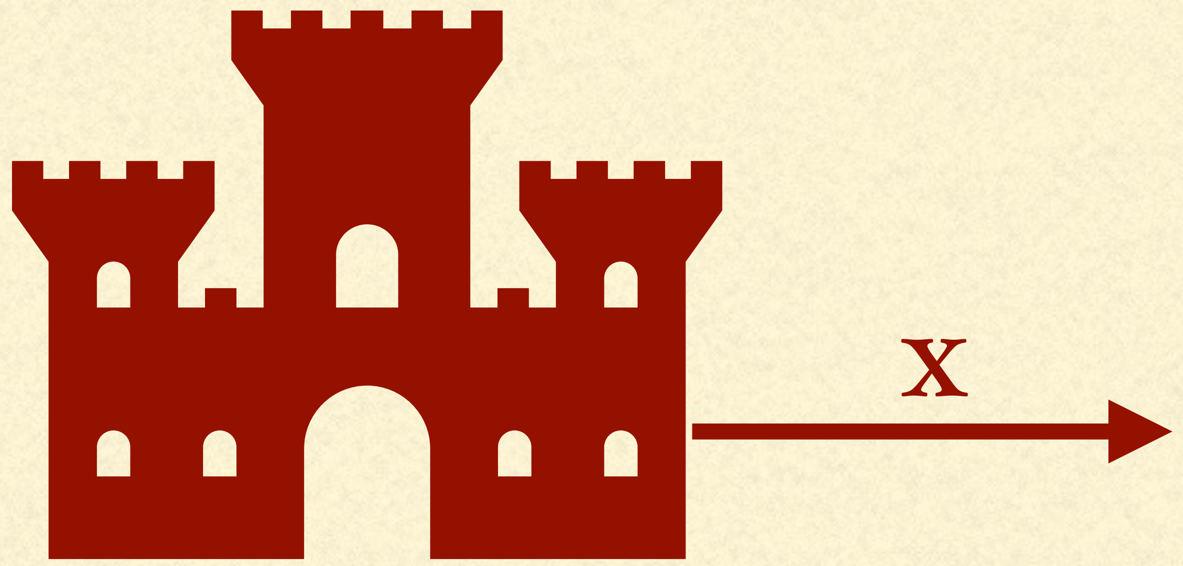
Suppose the Universe is in a momentum eigenstate.

Can I find an object in a superposition of momentum?

Does Preparer exist in Nature?

Suppose the Universe is in a momentum eigenstate.

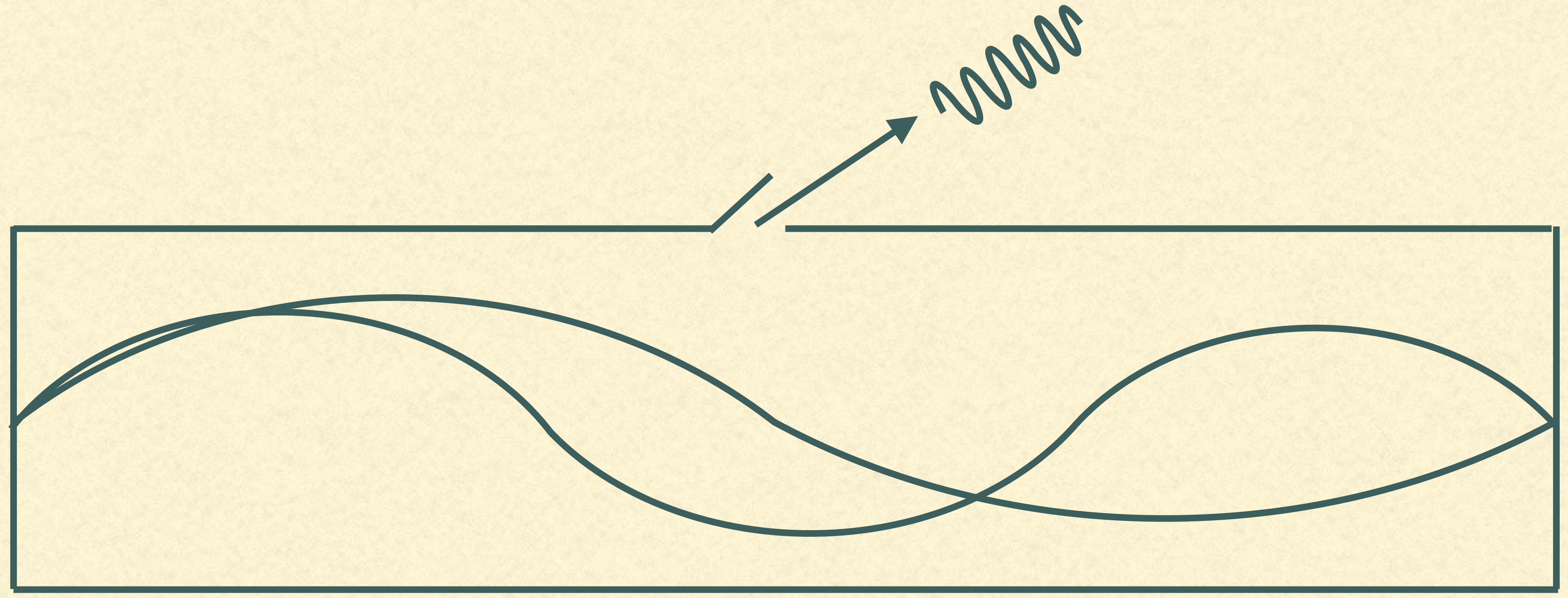
Can I find an object in a superposition of momentum?



Yes: we see many objects we can use to define a frame of reference for a position.

High Energy Paradox

Y. Aharonov, S. Popescu and D. Rohrlich, “Conservation laws and the foundations of quantum mechanics”.

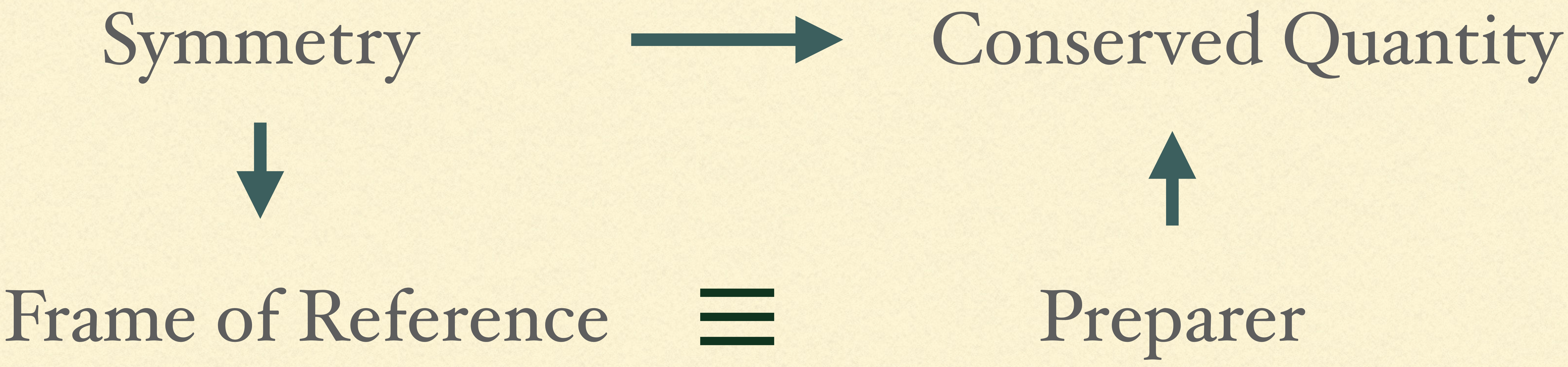


Photon in Superposition of Low Energies —Low Probability→ High Energy Photon

Noether's Theorem

Symmetry → Conserved Quantity

Noether's Theorem v2



Summary

* Total of a conserved quantity, angular momentum on a circle, is unchanged for each individual outcome of a measurement.
* Total across System & Preparer.
* Preparer is finite - no need to include the whole universe.
* Need a frame of reference to define an angle: this *is* the preparer.
* Paper on arXiv: D Collins and S Popescu, arXiv.2404.18621 (2024)

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* **Quantum Frame of Reference:** Y. Aharonov and L. Susskind, Phys. Rev. 155, 1428 (1967); S. D. Bartlett, T. Rudolph, and R. W. Spekkens, Rev. Mod. Phys. 79, 555 (2007).

* **Conservation Laws in Many Worlds:** L. Vaidman, PhilSci Archive Preprint 23509 (2024).

Appendix: Preparation in Angular Basis

$$
\begin{aligned}
|\phi_p\rangle_P |\hat{\mathbf{L}}_s = 0\rangle_S &= \sum_{l=-\infty}^{\infty} e^{-i\phi_p l} |l\rangle_P |\hat{\mathbf{L}}_s = 0\rangle_S \\
&\to \sum_{l=-\infty}^{\infty} e^{-i\phi_p l} \sum_{m=-\infty}^{\infty} \Psi(m) |l-m\rangle_P |m\rangle_S \\
&= \sum_{m=-\infty}^{\infty} e^{-i\phi_p m} \Psi(m) |\phi_p\rangle_P |m\rangle_S \\
&= e^{-i\phi_p \hat{\mathbf{L}}_s} |\phi_p\rangle_P \sum_{m=-\infty}^{\infty} \Psi(m) |m\rangle_S \\
&= e^{-i\phi_p \hat{\mathbf{L}}_s} |\phi_p\rangle_P \int \tilde{\Psi}(\phi_s) |\phi_s\rangle_S \, d\phi_s \\
&= |\phi_p\rangle_P \int \tilde{\Psi}(\phi_s) |\phi_s + \phi_p\rangle_S \, d\phi_s = |\phi_p\rangle_P \int \tilde{\Psi}(\phi_s - \phi_p) |\phi_s\rangle_S \, d\phi_s
\end{aligned}
$$

Appendix: The Grand-Preparer

Grand-Preparer starts in $|\Phi_g\rangle_G$, Preparer in $|0\rangle_P$, System in $|0\rangle_S$.

Interaction for preparing the Preparer:

$$|l\rangle_G|0\rangle_P \rightarrow \sum_{k=-\infty}^{\infty} \Phi_p(k)|l-k\rangle_G|k\rangle_P$$

Interaction for preparing the System:

$$|l\rangle_P|0\rangle_S \rightarrow \sum_{m=-\infty}^{\infty} \Psi(m)|l-m\rangle_P|m\rangle_S$$

Appendix: The Grand-Preparer 2

Prepare the Preparer:

$$\sum_{k=-\infty}^{\infty} \Phi_g(k)\,|k\rangle_G\,|0\rangle_P\,|0\rangle_S \rightarrow \sum_{k,l=-\infty}^{\infty} \Phi_g(k)\,\Phi_p(l)\,|k-l\rangle_G\,|l\rangle_P\,|0\rangle_S$$

Prepare the System:

$$\rightarrow \sum_{k,l,m=-\infty}^{\infty} \Phi_g(k)\,\Phi_p(l)\,\Psi(m)\,|k-l\rangle_G\,|l-m\rangle_P\,|m\rangle_S$$

Measure the System, outcome l_0:

$$\rightarrow \sum_{k,l=-\infty}^{\infty} \Phi_g(k)\,\Phi_p(l)\,|k-l\rangle_G\,|l-l_0\rangle_P\,|l_0\rangle_S$$